Evolutionary Grids of Interacting Stellar Binaries Containing Neutron Star Accretors

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High-performance computing has enabled astrophysicists to carry out detailed calculations of enormously complex systems. In this paper we describe the implementation of one such calculation involving the evolution of close, interacting binary stellar systems. In particular, we examine the evolution of systems containing an ordinary low-mass star that is being cannibalized by its compact companion (neutron star). Since there are many dimensions of parameter space associated with this problem (i.e., chemical composition, angular momentum dissipation, initial masses of the donor and the companion, chemical profile of the donor at the onset of mass transfer), using a highly efficient stellar evolution code that incorporates all of the salient physics (e.g., nuclear burning, hydrodynamics, radiative and convective transfer, tidal and rotational distortion) is essential. The results of the current investigation are briefly presented and future objectives for this research are also discussed.

Le calcul de haute performance a permis aux astrophysiciens de simuler des systèmes extrêmement complexes. Dans ce travail, nous décrivons la simulation de l'évolution d'un système binaire rapproché. Plus précisément, nous étudions l'évolution d'un système comportant une étoile ordinaire de faible masse cannibalisée par un compagnon compact (une étoile à neutrons). Puisque l'espace des paramètres associés à ce problème est vaste (composition chimique, dissipation du moment cinétique, masses initiales du donneur et de son compagnon, profil chimique du donneur au moment du transfert de masse), il est crucial d'appliquer un code d'évolution efficace qui incorpore toute la physique pertinente (combustion nucléaire, hydrodynamique, transport par rayonnement et convectif, distortions dues aux marées et à la rotation). Les résultats de notre recherche actuelle sont présentés, et les objectifs futurs sont discutés.

1 Introduction

Before the advent of computers, stellar structure and evolution was studied by purely analytic (and semi-analytic) methods ([1] [2]). However, due to the inherent nonlinearity of the equations of stellar structure and evolution, these approaches were extremely limited and it became necessary to use sophisticated numerical methods to obtain detailed quantitative information. In the modern era of stellar evolution, stellar codes are designed to be either: (i) highly efficient so as to accommodate analyses requiring large numbers of computations; or, (ii) extremely accurate so that stellar observations can be used to infer precise details concerning the physical properties of stars. In the latter case, helioseismology requires extremely precise theoretical models describing the modes of vibration of the Sun so that its internal structure can be deduced (e.g., the location of the boundary between the radiative core and the convective envelope [3]). These techniques have also been extended to the study of the interior properties of white dwarf stars. On the other hand, very efficient codes are needed to carry out population syntheses and to investigate problems whose 'parameter space' is extensive. In a typical population synthesis, the main objective is to follow the evolution of a population of stars that is generated using Monte Carlo techniques. For example, the evolution of a globular cluster containing a million stars can be followed by calculating the evolution of each star individually and taking into account its possible interactions with other stars in the cluster. Parameter space analyses are conducted so that the widest spectrum of results corresponding to a physically realistic range of parameters (e.g., the initial chemical composition of star) can be generated. This type of analysis can also be used to develop a general picture of a particular physical phenomenon associated with the evolution of stars for which certain physical parameters are not well known.

1.1 Interacting Binaries

Interacting binary systems are some of the most exotic and intriguing astronomical objects in the Galaxy. They are composed of two stars that are in such close proximity that at least one of them is affecting the evolution of the other. In most cases this interaction is manifested in the form of mass transfer (e.g., hydrogen-rich gas from the atmosphere of the donor star can be pulled off by its accreting companion). If the stars are very close to each other then the donor star will be very tidally and rotationally distorted. The actual shape of the donor depends on the geometry of the equipotential surfaces (i.e., Roche geometry [4]). When the donor fills its critical Roche lobe, matter will flow from its atmosphere through the inner Lagrange point and eventually settle on the surface of the accretor (the inner Lagrange point is a point of unstable gravitational equilibrium that is located directly between the centers of the two stars).

The most important subclass of these interacting binaries contains an accreting compact object (e.g., a white dwarf, neutron star, or black hole). Since the gas that flows through the inner Lagrange point has non-zero angular momentum, it cannot fall directly onto the surface of the accretor. Instead it spirals in towards the accretor in the form of an accretion disk (see Figure 1). Strongly magnetic white dwarfs are the only exceptions to this mode of mass trans-

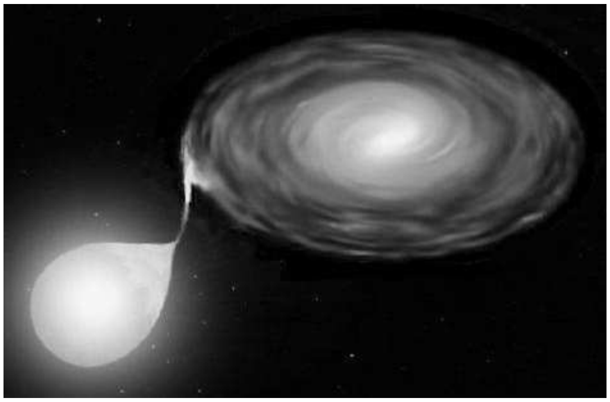

Figure 1. Artist's depiction of an interacting binary. Note the tear-drop shape of the donor star resulting from tidal and rotational distortion. Gas stripped from the donor forms an accretion disk. The neutron star at the center of the accretion disk is too small to be seen.

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Interacting binaries containing compact objects are intrinsically bright because of the great depth of their gravitational potential wells (this is due to the fact that much of the gravitational potential energy of the infalling matter is converted to electromagnetic radiation). Partial eclipses and disk instabilities lead to changes in the intensity of the light observed from these systems and hence they are often referred to as "Variable Stars". Periodicities in the light curves allow astronomers to deduce certain properties such as their orbital periods. Systems composed of white dwarf accretors (about the same size as the Earth but extremely dense) that are cannibalizing ordinary stars (similar to our Sun), are known as Cataclysmic Variables. They are of particular interest because they may be the progenitors of Type Ia supernovae (extremely violent and bright cosmic explosions). These supernovae are used extensively by astronomers to determine the distance scale to the farthest observable points in the Universe. In conjunction with other observations, the supernova measurements have only recently allowed astronomers to conclude that the expansion of the Universe is presently accelerating! This conclusion has profound implications for our understanding of what dominates the evolution of the Universe (i.e., the 'dark energy' problem [5]).

1.2 Low-Mass X-Ray Binaries (LMXB's)

Low-Mass X-ray Binaries are interacting systems wherein a low mass star ($< 3M_\odot$) is being cannibalized by a neutron-star accretor (see [6], and references therein). Because of the depth of the gravitational potential well of the neutron star, the photons emitted from these systems are very energetic and thus these systems are typically discovered at X-ray frequencies. LMXB's can be formed if one of the stars in the progenitor binary undergoes a supernova explosion (assuming that the explosion does not disrupt the binary). The star that explodes leaves behind a neutron star remnant and it becomes the compact accretor. Once the donor star attaches to its (critical) Roche lobe, mass transfer ensues and an accretion disk forms around the neutron star. According to the conventional model, the neutron star is spun up by accretion torques causing it to become a pulsar.

Depending on the timescales for orbital angular momentum dissipation (e.g., gravitational radiation reaction, or magnetic stellar wind braking) and nuclear evolution (i.e., nuclear burning in the core of the donor star), the binary can either evolve to extremely short orbital periods (perhaps even as small as several minutes) or evolve to very long periods (thousands of days). Nuclear evolution causes the donor star to expand and become a red giant (with concomitantly long orbital periods) while angular momentum losses tend to enhance mass transfer which leads to an attenuation in nuclear burning. The dividing line between LMXB's that evolve to extremely short periods and those that evolve to long periods is known as the *bifurcation limit* and is very sensitive to the assumed input physics ([7]). Above the bifurcation limit, the physical radius of the donor is largely governed by its core mass; that is, the mass of the nearly pure helium core created as the result of hydrogen being fused into helium (i.e., hydrogen burning). As the donor evolves (simultaneously losing mass and growing larger in radius) it reaches a point where so much mass has been transferred that only a small amount of hydrogen surrounds its core. At this juncture, the donor collapses (i.e., detaches from its Roche lobe) and mass transfer ceases. What remains is a low-mass helium degenerate dwarf [HeDD] in orbit with a (recycled) pulsar. In the past decade more than fifty of these systems have been discovered ([8]). It is now clear that LMXB's that evolve above the bifurcation limit are the progenitors of many of these binary millisecond pulsar systems.

But what about LMXB's that evolve below the bifurcation limit? Only very recently have pulsars been discovered in accreting binary systems (i.e., SAX J1808.4-3658[9]; XTE J0929-314 [10]; XTE J1751-305[11]; XTE J1807-294 [12]), and all have ultrashort orbital periods (between 35 minutes and 2 hours). These discoveries fill in the 'missing link' in terms of our understanding of the origin of recycled pulsars and seem to confirm our theoretical models of the evolution of LMXB's. The calculations that are presented in this paper verify the basic model (over a very large portion of parameter space) while providing us with a detailed picture of the physical phenomena associated with the evolution of LMXB's (e.g., hydrogen shell flashes in the envelope of the donor).

2 Methodology

The evolution of interacting binaries containing compact objects is largely governed by the response of the donor to the orbital dynamics of the binary. Solving for this response is the difficult part of the computation. Based on semi-analytic hydrodynamic rules for the rate of mass transfer as a function of the location of the Roche lobe relative to the surface of the donor, it is possible to decouple the evolution of the orbit (and accretion of a matter onto the compact object) from the evolution of the donor itself.

2.1 Evolution of the Donor

Ignoring the effects of magnetic fields, the equations of stellar structure can be expressed in terms of four partial differential equations that describe the hydrodynamic evolution, mass continuity, energy conservation, and mode of energy transfer (radiative, conductive, and/or convective transfer) inside the donor star. A Lagrangian approach is adopted such that the variable m_r defines the mass contained inside a sphere of radius r, and t denotes the elapsed time. These equations can be expressed as:

$$\begin{aligned}
\frac{\partial P}{\partial m_r} &= -\left(\frac{G}{4\pi}\right)\frac{m_r}{r^4} - \frac{1}{4\pi r^2}\left(\frac{\partial^2 r}{\partial t^2}\right) \\
\frac{\partial r}{\partial m_r} &= \frac{1}{4\pi r^2 \rho} \\
\frac{\partial L}{\partial m_r} &= \varepsilon_{nuc} - \varepsilon_\nu - T\left(\frac{\partial S}{\partial t}\right) \\
\frac{\partial T}{\partial m_r} &= -\left(\frac{3}{64\pi^2 ac}\right)\frac{L\kappa}{r^4 T^3} \quad \text{[Rad/Cond]} \\
\frac{\partial T}{\partial m_r} &= \left(\frac{T}{P}\right)\left(\frac{\partial P}{\partial m_r}\right)\nabla_{ad} \quad \text{[Convective]} \qquad (1)
\end{aligned}$$

where P is the pressure, ρ is the mass density, T is the temperature, S is the specific entropy, L is the luminosity, ϵ_{nuc} is the rate of nuclear energy generation (per unit mass), ϵ_ν is the rate of energy losses due to neutrinos (per unit mass), κ is the opacity (for radiative/conductive transfer), and ∇_{ad} is the adiabatic temperature gradient (for convective transfer). Note that G, a, and c are physical constants.

2.1.1 *Input Physics*

In order to solve the equations of stellar evolution, the input physics must be specified. An equation of state (EOS) must be adopted, and the dependence of nuclear energy generation (ε_{nuc}), neutrino losses (ε_ν), and the radiative/conductive opacities on the density, temperature and chemical composition must be specified. These can be stated (respectively) as:

$$\begin{aligned}
P &= P(\rho, T, X_i) \\
\varepsilon_{nuc} &= \varepsilon_{nuc}(\rho, T, X_i) \\
\varepsilon_\nu &= \varepsilon_\nu(\rho, T, X_i) \\
\kappa &= \kappa(\rho, T, X_i) \qquad (2)
\end{aligned}$$

where X_i is the abundance vector (i.e., the abundance of the i^{th} chemical constituent). Additional constraints associated with the creation and destruction of the chemical elements due to nuclear burning and particle decays must also be specified.

The input physics that was used in these computations is described extensively in a paper by Nelson, MacCannell, & Dubeau ([13]; and references therein). Specifically, we implemented the OPAL radiative opacities ([14]) in conjunction with the low-temperature opacities of Alexander & Ferguson ([15]), and the Hubbard & Lampe ([16]) conductive opacities. Great care was taken to ensure that each of these opacities blends smoothly across their respective boundaries of validity. Our treatment enforces continuity of the respective first-order partial derivatives over the enormous range of the independent variables (i.e., ρ, T, and X_i) that are needed to fully describe the properties of the donors. The equation of state is a combination of an analytic EOS for arbitrary mixtures of hydrogen, helium, and heavier elements that is matched to the Magni & Mazzitelli ([17]) EOS for pure helium. The analytic EOS includes the effects of molecular hydrogen formation, radiation, arbitrarily relativistic electron degeneracy, and an approximate treatment of pressure ionization. The effects of plasma neutrino cooling are calculated according to the prescription of Itoh ([18]). Nuclear energy generation rates are calculated based on the work of Fowler et al. ([19]).

2.1.2 *Method of Solution*

The four equations of stellar structure are solved using a variant of the Henyey method ([20]). In essence, a mesh is superimposed on the star that effectively divides it into a number of zones (typically between several hundred to nearly 5,000 depending on the run of the physical variables). The differential equations are then written in finite difference form and solved using a 'relaxation technique' under the constraints imposed by the four (central and surface) boundary conditions. In order to avoid numerical singularities at the center ($r = 0$ and $L = 0$), the differential equations of the central zone are solved using high-order series solutions. To increase global accuracy, a small outer zone containing the stellar atmosphere and approximately $\sim 0.01\%$ of the stellar mass is calculated *a priori* and is used to provide the outer boundary conditions (in terms of T and P) for the solution of the linearized system of difference equations.

This technique requires an initial guess for the internal structure of the star (i.e., a 'block start'). The values of the physical variables must be specified at the edges of each zone and midzone values are interpolated. Once a set of approximate solutions has been applied, corrections to the physical variables can be computed thereby producing a more exact solution. This process is repeated through many successive iterations until the desired level of accuracy is obtained. An adaptive mesh is employed so that mass transfer can be handled smoothly. Since a temporal sequence of models is being calculated, the results from the previously computed model (at $t = t_o$) can be used as an initial approximation for the structure of the new model ($t_o + \delta t$). The physical structure is then allowed to relax to an arbitrarily precise solution.

2.2 Parameter Space

The parameter space needed to fully investigate the evolution of this type of binary system is inherently five dimensional. The vector of parameter space contains the following elements: (i) the initial mass of the donor (M_2); (ii) the initial mass of the accretor (M_1); (iii) the initial chemical profile of the interior of the donor star; (iv) the metallicity of the donor (i.e., the concentration of elements heavier than helium [Z]); and, (v) the mode of angular momentum losses that drives mass transfer. Thus a grid of representative initial conditions needs to be created and each individual case must be evolved. The resolution of the grid is simply constrained by the available computing power. A minimal grid, however, should comprise at least 120 unique cases (using the five dimensional vector defined above, this number is calculated as $2 \times 2 \times 10 \times 3 \times 1$, respectively). Each time sequence (track) in the evolution requires that between 10^5 and 5×10^5 models be calculated. Time steps as small as several minutes or as large as one million years are employed. The actual value of the time step depends on the physical timescales that are relevant for each phase of the evolution. For example, long phases of slow cooling can be calculated using large time steps while nuclear shell flashes near the surface of the donor star require very short time steps. Since there are several physical variables that are defined at each of the internal mesh points, our 'minimal grid' generates approximately 10 terabytes of data of which only approximately 1% is saved.

Because the calculation is characterized by many unique initial conditions, our computational strategy is to assign a different set of initial conditions to each individual processor. We have carried out these calculations in a number of different ways in order to optimize the available computing resources. Originally we used PVM and our own distribution and retrieval software to carry out computations on approximately 100 NT-based Intel platforms (200 - 1800 MHz) that are located in various labs located throughout Bishop's University. We have also used a Beowulf cluster (36 Intel CPU's running at 1.8 to 2.4 GHz) that is located at the Université de Sherbrooke (Centre de Calcul Scientifique). At 1.8 GHz, a single evolutionary sequence can generally be completed within a month of dedicated CPU time. We are in the process of constructing a 128 node cluster that will be combined with the existing cluster. This increase in computing power should allow us to easily enlarge the grid size by at least an order of magnitude.

3 Results

Stars are born as the result of the fragmentation of giant interstellar molecular cloud complexes (composed mostly of hydrogen). Each fragment can collapse under its own gravity and this causes the hydrogen gas to heat up. Once the temperature is sufficiently high ($\sim 10^7$ K), nuclear fusion of hydrogen into helium will commence. A star that has just started to burn hydrogen and is in approximate thermal equilibrium is said to be a Zero Age Main Sequence (ZAMS) star. Its hydrogen abundance is approximately uniform throughout its stellar interior. For the calculations presented in this paper, the initial mass fraction of hydrogen of stars on the ZAMS is taken to be $X = 0.71$. As the star evolves, its hydrogen abundance decreases most quickly at its center where temperatures and densities are highest. When its central hydrogen abundance reaches zero (i.e., $X_c = 0$), the star is said to be on the Terminal Age Main Sequence (TAMS). Past this point, a low mass star develops a helium core and enters the subgiant phase. Once the core has attained a mass of $M_c \simeq 0.20 M_\odot$, it becomes a red giant and its radius grows very quickly.

In order to investigate the effects of different chemical profiles on the evolution of these interacting binary systems, we allowed single stars to evolve from the ZAMS to the tip of the red giant branch (i.e., just before the ignition of helium burning). The star (i.e., the donor) was then brought into contact with its Roche lobe at various points during its chemical evolution (expressed in terms of X_c and M_c). At the *onset of mass transfer*, the initial conditions corresponded to $X_c = 0.71, 0.3, 0.1$ for stars without cores and $M_c = 0$ to $0.35 M_\odot$ (in increments of $0.05 M_\odot$) for stars that had developed cores. Note that if the neutron star and donor are very close to each other after the binary has been formed, the donor will not have had much time to evolve before filling its (critical) Roche lobe and thus its value of X_c will be very large when mass transfer first occurs. However if they are formed with a relatively wide separation, the donor may have to become a giant (with a large core mass) by the time that it fills its Roche lobe. Thus the initial chemical profile of the donor is correlated with the initial orbital separation of the two stars.

We calculated approximately 120 evolutionary tracks for the parameter space described in §2.2. The initial masses of the donors corresponded to values of $M_2 = 1$ and $1.5 M_\odot$, while the masses of the accretors were taken to be $M_1 = 0.7$ and $1.4 M_\odot$ (the latter value being regarded as the 'canonical' neutron star mass). The initial chemical profile of the donor was parameterized in terms of either X_c or M_c, while the donor's metallicity was chosen to be representative of Population I and II stars. Specifically we adopted values of $Z = 0.0001$, 0.004, and 0.02. Note that $Z = 0.02$ is approximately equal to the (Population I) solar value.

With regard to the final dimension of parameter space, it was assumed that gravitational radiation and magnetic stellar wind (MSW) braking were responsible for orbital angular momentum losses. We used the RVJ ([21]) parameterization of the Verbunt-Zwaan MSW braking law (i.e., $\dot{J}_{orb} \propto R^\gamma$ where $\dot{J}_{orb}$ is the time derivative of the orbital angular momentum, R is the radius of the donor star and γ is a dimensionless number). For the present investigation, we set $\gamma = 3$. Mass transfer was assumed to be completely non-conservative. We thus set the mass-capture fraction (i.e., $|\dot{M}_1|/|\dot{M}_2|$) equal to zero, and assumed that the matter lost from the system carried away a specific an-

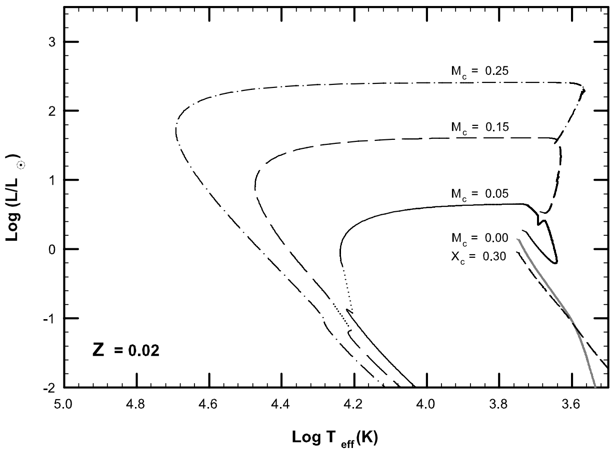

Figure 2. Evolutionary tracks of various interacting binary systems plotted on an HR diagram. For each track, the initial mass of the donor is $1.0M_{\odot}$, its metallicity is $Z = 0.02$, and the mass of the accreting neutron star is $1.4M_{\odot}$. The symbols X_c and M_c denote the values of the central hydrogen content and the helium core mass (in solar masses) of the donor at the onset of mass transfer, respectively. Note that $M_c = 0.0$ is equivalent to $X_c = 0$.

gular momentum equal to that of the neutron star (i.e., fast Jeans' mode). There are several other possible scenarios that we plan to investigate, including conservative mass transfer ($|\dot{M}_1| = 0$), and using other values of γ.

3.1 Evolutionary Tracks

Many physical variables are needed to fully describe the properties of evolving binary systems. These include the orbital period (P_{orb}), mass transfer rate, masses of the two stars, chemical profile of the interior of the donor, and its surface luminosity (L) and effective surface temperature (T_e). The luminosity and effective temperature are very important quantities in stellar astronomy since both can, in principle, be measured if the distance to the star is known. Moreover, the radius can be related to these quantities using the blackbody relation $L = 4\pi\sigma R^2 {T_e}^4$. A plot of $\log L$ versus $\log T_e$ is known as the Hertzsprung-Russell (HR) diagram and the path that a star follows in this diagram is known as its evolutionary track. In Figure 2 we present the evolutionary tracks of a $1.0M_{\odot}$ metal-rich ($Z = 0.02$) donor and a $1.4M_{\odot}$ accretor in the HR diagram for various initial states of chemical evolution. The donors having initial values of $X_c = 0.3$ and 0 (the latter being equivalent to $M_c = 0$) exhibit very different evolutionary tracks compared to donors that have developed helium cores. The dividing line between these two types of evolution is known as the bifurcation limit. For these two cases (below the limit), the donors lose mass and evolve towards increasingly lower luminosities and effective temperatures. Their radii continuously decrease leading to a decrease in P_{orb} (this behavior changes when their masses are $< 0.1M_{\odot}$ and electron degeneracy dominates the EOS). For the two systems, the initial value of P_{orb} is ~ 10 hours and it decreases to values of < 1 hour. Even though the $X_c = 0$ donor is trying to build a core, mass is lost at a sufficiently high rate that nuclear burning (and further growth of the core) is stymied.

For larger core masses ($\geq 0.05M_{\odot}$) the donor does evolve through the subgiant phase and becomes a red giant (this corresponds to the somewhat vertical parts of the tracks seen in Figure 2). For many of these donors, their luminosities increase by more than an order of magnitude while simultaneously losing more than one-half of their mass. Eventually so much mass is stripped away from the donor that its mass becomes comparable to the mass of the core itself. The remaining hydrogen-rich envelope contains so little mass that it can no longer provide sufficient pressure to sustain hydrogen burning. Thus the envelope collapses and the radius of the donor star decreases substantially. This critical juncture in the evolution corresponds to the nearly horizontal parts of the tracks seen in Figure 2. During this phase their radii can drop between one to two orders of magnitude. This isoluminous phase has a very short duration ($< 10^6$ years) compared with the billions that it took for the donors to reach this stage. Once a donor reaches its maximum effective temperature, there is a precipitous decrease in the (surface) luminosity. The donor subsequently consists largely of a helium core ($0.15 \leq (M/M_{\odot}) \leq 0.45$) on top of which exists an extremely thin hydrogen envelope ($< 10^{-3}M_{\odot}$). As the donor continues to collapse, the hydrogen is reheated and a "thermonuclear flash" ensues. Depending on the mass of the core, the ignition of hydrogen burning can be quiescent or extremely violent (hydrodynamic) ([22]). Moreover, once the hydrogen re-ignites, the donor can expand and fill its Roche lobe causing further mass transfer. Eventually the thermonuclear runaway is quenched and the donor's envelope collapses. It is possible for the donor to go through several cycles like this before nuclear burning is permanently extinguished. After the final flash, the donor is destined to cool and approach its zero-temperature configuration (i.e., become completely electronic degenerate). Such a star is referred to as a helium degenerate dwarf [HeDD]. The typical radii of these very cool and faint dwarfs is only a few hundredths of a solar radius. The cooling tracks are nearly straight lines corresponding to approximately constant radii (but decreasing luminosities and temperatures). Also note in Figure 2 that the dotted lines denote the portions of a particular evolutionary track *between* which violent thermonuclear flashes occur (e.g., the 0.05 and $0.15M_{\odot}$ cases).

An example of a two cycle hydrogen shell flash is shown in Figure 3. The initial conditions correspond to a $1M_{\odot}$ donor with an initial core mass of $0.1M_{\odot}$ that is losing mass to a $1.4M_{\odot}$ accretor ($Z = 0.02$). Once the donor climbs the red giant branch, it reaches the isoluminous phase and then contracts until the hydrogen envelope is sufficiently hot causing nuclear burning to be re-ignited. The luminosity of the donor rises temporarily and then eventually returns to a point where the process begins again.

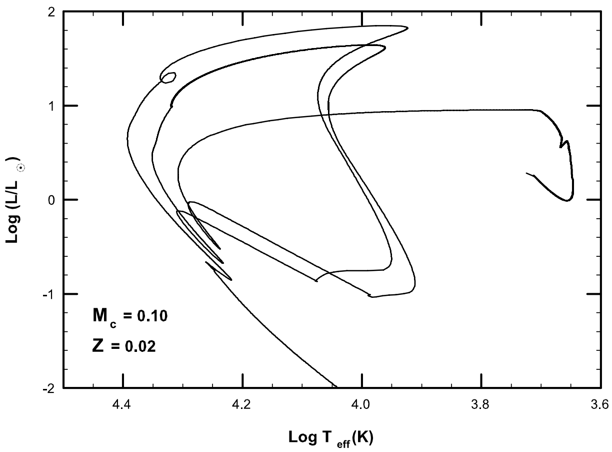

Figure 3. The evolution of an initial $1.0M_{\odot}$ donor ($Z = 0.02$) in orbit with a $1.4M_{\odot}$ accreting neutron star plotted on the HR diagram. The presence of two violent hydrogen shell flashes is evident from the two large loops that occupy the left-hand side of the diagram.

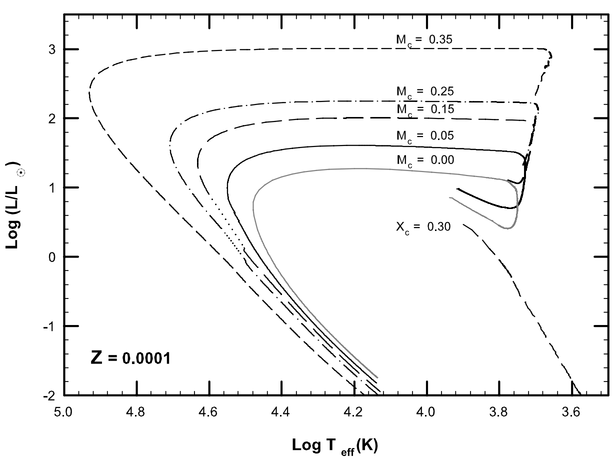

Figure 4. Evolutionary tracks of various interacting binary systems plotted on an HR diagram. For each track, the initial mass of the donor is $1.0M_{\odot}$, its metallicity is $Z = 0.0001$, and the mass of the accreting neutron star is $1.4M_{\odot}$. The symbols X_c and M_c denote the values of the central hydrogen content and the helium core mass (in solar masses) of the donor at the onset of mass transfer, respectively. Note that $M_c = 0.0$ is equivalent to $X_c = 0$.

The third time that the donor reaches this point, the density of the hydrogen in the shell is sufficiently low that only a small flash occurs and then the donor settles into a very long stage of cooling.

In Figure 4 we present the evolutionary tracks of extremely metal-poor donors ($Z = 0.0001$). These stars are associated with the very first generation of stars that was formed in our galaxy (the hydrogen gas clouds from which they formed had not yet been contaminated by heavy elements produced by supernova explosions). The behavior of the evolutionary tracks is very similar to that seen for $Z = 0.02$ but we notice that, unlike the previous case, a donor with an initially zero core mass can become a giant and ultimately produce a HeDD. Thus the location of the bifurcation limit is different. Stars with lower metallicities are more luminous and burn their supplies of hydrogen faster. Consequently they have a higher probability of forming a helium core before mass loss can extinguish nuclear burning. Note the existence of violent thermonuclear runaways for $M_c = 0.15$ and $0.25M_{\odot}$.

4 Conclusion

The large-scale numerical computations presented in this paper have provided us with realistic models describing the evolution of interacting binary systems containing neutron star accretors. Although parameter space is five dimensional, we have been able to explore a large slice of it. The grid of initial conditions was chosen so as to cover the most realistic range of physical parameters. The models allow us to conclude that there is an extremely sharp bifurcation between systems for which the donors evolve to become giants and then subsequently cool as HeDD's, and those leading to continuous mass loss wherein the donors are eventually reduced to Jupiter-sized objects ($\sim 0.001M_{\odot}$). Astronomers have observed the end-products of both evolutionary outcomes. The binary pulsar system PSR B1855+09 is a typical example of a LMXB that has evolved to yield a millisecond pulsar in orbit with a HeDD. Using a radio telescope to measure the Shapiro delay of the pulsar signal, the mass of the HeDD has been measured to be $0.258M_{\odot}$ to within $\sim 0.02M_{\odot}$ ([23]). The observational results are in the good agreement with our evolutionary models and we can conclude that, based on their estimate of the effective temperature (4800 K), the HeDD is likely to have a luminosity of $\approx 2 \times 10^{-4}L_{\odot}$ and an age (after the onset of mass transfer) of $\approx 12.5 \times 10^{9}$ years.At the other side of the bifurcation limit, X-ray pulsars in accreting binary systems have just recently been discovered. These systems (e.g., XTE J0929-314) have very short orbital periods and this is exactly what the theoretical models predict for binary systems just below the bifurcation limit (more detailed results concerning accreting binary millisecond pulsar systems can be found in the work of Nelson and Rappaport ([24]).

To make more progress in understanding the evolution of these systems we must first fully explore parameter space and then conduct the appropriate population synthesis analysis. The population synthesis is an enormous undertaking since the first stage of the synthesis may produce thousands of LMXB's and all of their evolutions will have to be computed. A sufficiently large population synthesis should allow us to determine the probabilities of the formation of the various types of binary pulsars that are observed in the galaxy, and this in turn should allow us to constrain the mode of angular momentum dissipation in these systems. We are currently in the process of undertaking this investigation. All of the data presented in this paper (in addition to specific requests) can be obtained from the Web server located at: **physics.ubishops.ca/evolution**

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References

1. S. Chandrasekhar. *An Introduction to the Study of Stellar Structure*. New York: Dover, 1938.
2. M. Schwarzschild. *Structure and Evolution of the Stars*. New York: Dover, 1958.
3. J.N. Bahcall, M.H. Pinsonneault, and S. Basu. *Astrophysical Journal*, 555:990, 2001.
4. Z. Kopal. *Close Binary Systems*. New York: Chapman & Hall, 1959.
5. S. Perlmutter et al. *Astrophysical Journal*, 516:565, 1999.
6. P.C. Joss and S.A. Rappaport. *Annual Reviews of Astronomy and Astrophysics*, 22:537, 1984.
7. E. Pylyser and G.-J. Savonije. *Astronomy and Astrophysics*, 191:57, 1988.
8. F. Camillo, D.R. Lorimer, P Friere, A.G. Lyne, and R.N. Manchester. *Astrophysical Journal*, 535:975, 2000.
9. D. Chakrabarty and E.H. Morgan. *Nature*, 394:346, 1998.
10. D.K. Galloway, D. Chakrabarty, E.H. Morgan, and R.A. Remillard. *Astrophysical Journal (Letters)*.
11. C.B. Markwardt and J.H. Swank. *I.A.U. Circulars*, # 7867, 2002.
12. C.B. Markwardt, E. Smith, and J.H. Swank. *I.A.U. Circulars*, # 8080, 2003.
13. L.A. Nelson, E. Dubeau, and K.MacCannell. *Astrophysical Journal*, (submitted), 2003.
14. C. A. Iglesias and F. J. Rogers. *Astrophysical Journal*, 464:943, 1996.
15. D.R. Alexander and J.W. Ferguson. *Astrophysical Journal*, 437:879, 1994.
16. W. B. Hubbard and M. Lampe. *Astrophysical Journal (Supplement)*, 18:163, 1969.
17. G. Magni and I. Mazzitelli. *Astronomy and Astrophysics*, 72:134, 1979.
18. N. Itoh. Neutrino energy loss in stellar interiors. In T. Kajino, editor, *Proceedings of International Conference on Origin and Evolution of the Elements*. World Scientific, 1993.
19. W. Fowler, W.A. Caughlin, and B. Zimmermann. *Annual Reviews of Astronomy and Astrophysics*, 13:69, 1975.
20. R. Kippenhahn & A. Weigert. *Stellar Structure and Evolution*. Berlin: Springer-Verlag, 1991.
21. S. Rappaport, F. Verbunt, and P.C. Joss. *Astrophysical Journal*, 275:713, 1993.
22. Ph. Podsiadlowski, S. Rappaport, and E.D. Pfahl. *Astrophysical Journal*, 565:1107, 2002.
23. M.H. van Kerkwijk, J.F. Bell, V.M. Kaspi, and S.R. Kulkarni. *Astrophysical Journal (Letters)*, 530:37, 2000.
24. L.A. Nelson and S. Rappaport. *Astrophysical Journal*, (submitted), 2003.